

# DC Machines

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# Objective of Lecture

- Objective: to investigate the torque/speed characteristics of series and shunt DC motors and describe typical applications for both types

## Review

- There are two types of DC machine to be considered: series and shunt connected motors
- Series motors have the armature and field coils connected in series
- Shunt motors have the armature and field coils connected in parallel

# DC Motor Relationships

- In the motor armature, the effect of the applied voltage,  $V_a$ , is reduced by the influence of an induced emf,  $V_{emf}$ , which is established through the process of electromagnetic induction in opposition to the applied voltage

- Therefore the equivalent applied voltage,  $V_{eq}$ , equals

$$V_{eq} = V_a - V_{emf}$$

- The induced emf is only present when the motor armature is turning. If the motor is stopped then there is no induced emf, as no lines of flux are being cut, and the armature current will be a maximum (as  $V_{eq} = V_a$ ). Note that the induced emf can never be equal to or higher than the applied voltage

# DC Motor Relationships - contd

- As  $V_a$  is 'regulated' by the induced emf, the armature current is regulated, i.e. if no emf was induced the armature current flowing would be much higher and the speed of motor operation would be much higher
- The value of induced emf,  $V_{emf} \propto$  rate of change of flux, or
$$V_{emf} = \Phi\omega K_e$$
- where  $\Phi$  is the motor flux,  $\omega$  is the rotational speed of the motor in rads/sec and  $K_e$  is a constant relating motor parameters such as number of poles etc and is specific for a particular motor
- The flux,  $\Phi$ , is established by the field current  $I_f$

# DC Motor Relationships - contd

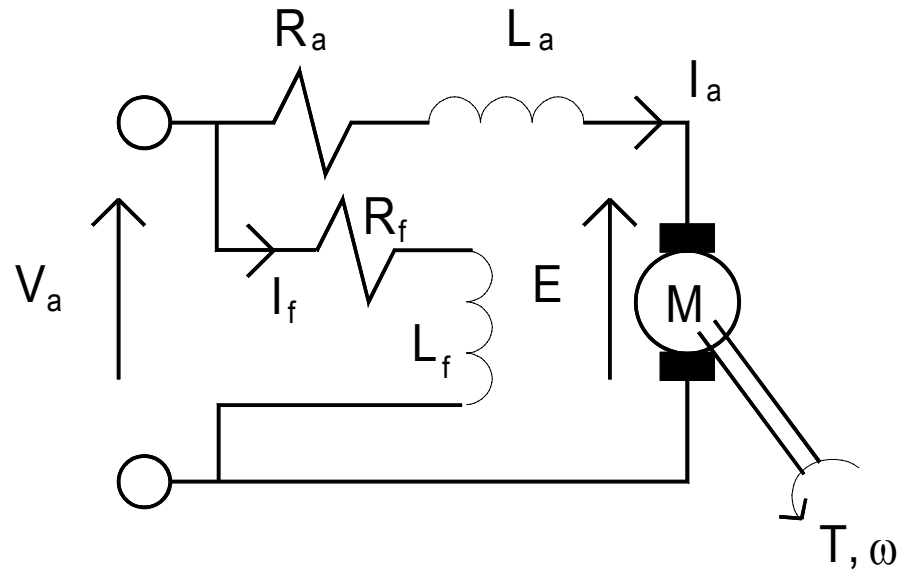
- The torque output of the motor is proportional to the force on the conductors and from  $F = BI_aL = (\Phi/A)I_aL$  then

$$T = \Phi I_a K_t$$

- where  $\Phi$  is as before,  $I_a$  is the armature current and  $K_t$  is the torque constant of the motor.

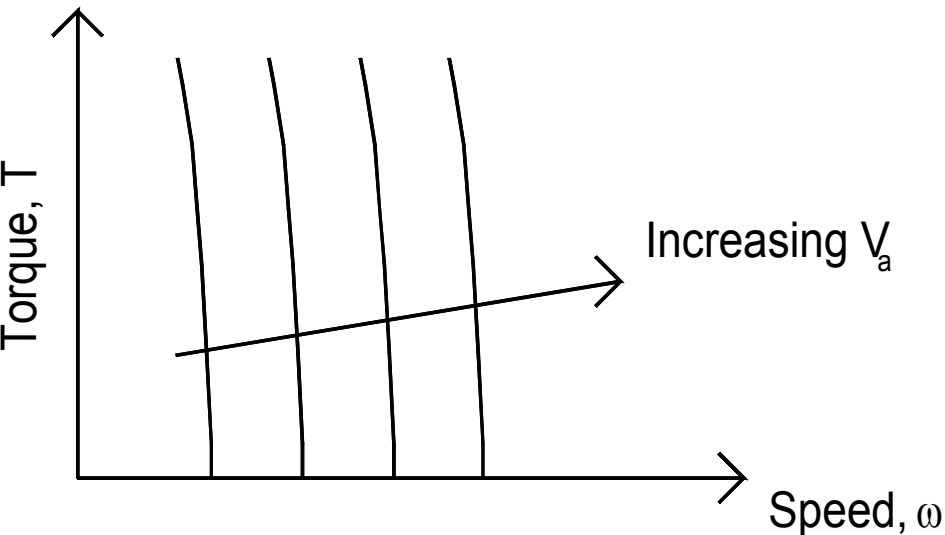
# DC Shunt Motor

- For a shunt connected motor if the applied voltage is constant then the field current,  $I_f$ , and therefore the flux,  $\Phi$ , is constant
- $E \propto \omega$  ( $E = V_{emf} = \Phi \omega K_e$ )
- So when a load is applied to the shunt motor the speed decreases, and therefore so does  $E$
- As  $E$  decreases then the armature current,  $I_a$ , starts to increase and the torque,  $T = I_f I_a K_t$ , starts to increase
- $V_a = I_a R_a + E \Rightarrow \omega = \frac{V_a - I_a R_a}{I_f K_e}$



# DC Shunt Motor

Torque vs Speed Characteristics  
for a DC Shunt Motor



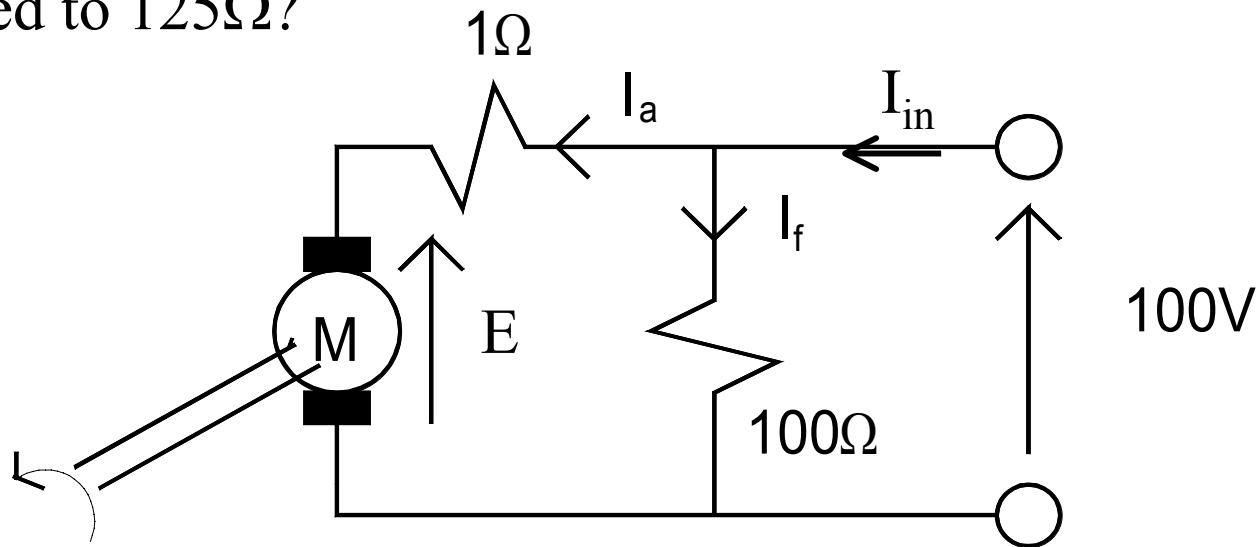
- The motor torque will increase until it matches the applied load torque
- For typical values of  $I_a$ , and  $R_a$ , it is normal to assume  $V_{emf} \sim V_a$
- Hence as  $V_a$  is varied so does  $E$  and  $\omega$
- Practically, at no-load ( $T \sim 0$ ),  $I_a$  is small and  $E \sim V_a$ , and as the load torque increases then  $I_a$  increases and causes  $E$  to fall slightly relative to  $V_a$ .
- Therefore the speed does fall slightly as torque increases but in general for a shunt motor the speed is considered constant w.r.t Torque

# Practical Concerns: Shunt Motor

- Field Weakening: Examine the earlier equation  $\omega = \frac{V_a - I_a R_a}{I_f K_e}$
- The speed could be varied by varying  $I_f$
- This is known as field weakening - however there is a danger if the field current is reduced to much - the motor could overspeed!!
- Some motors sense any decrease in field current and operate a protective relay trip i.e. disconnect the power
- $I_f$  would typically be varied by changing the field resistance  $R_f$

# Example: Shunt Motor

- Assume that the shunt motor is supplied from a 100V supply, and has an armature resistance of  $1\Omega$  and a field resistance of  $100\Omega$ , torque of  $0.949\text{Nm}$  at a speed of  $990\text{rpm}$  and  $K_e = 0.955 \text{ V}/(\text{A}\cdot\text{rad}/\text{s})$ .
- Calculate the power output of this motor, the efficiency at this load and the new speed if the field resistance is changed to  $125\Omega$ ?



# Example: Shunt Motor - contd

- Power Output,  $P = T\omega = T \cdot (2\pi/60) \cdot n$ , where  $T$  is the motor torque and  $n$  is the motor speed in rpm
- $P = 0.949 \cdot (2\pi/60) \cdot 990 = 98.4 \text{ W}$
- Power input =  $I_{in} \cdot V_{in} = (I_a + I_f) \cdot V_{in}$ . The field current can be calculated easily from the circuit diagram  $I_f = 100\text{V}/100\Omega = 1\text{A}$ .
- $E = \phi\omega K_e = I_f\omega K_e = 1 \cdot (2\pi/60) \cdot 990 \cdot 0.955 = 99\text{V}$
- $V_a = I_a \cdot R_a + E \Rightarrow I_a = (100 - 99)/1 = 1\text{A}$
- Therefore Power input =  $(1 + 1) \cdot 100 = 200\text{W}$
- The motor efficiency is  $\eta = P_o/P_{in} \cdot 100\% = 98.4/200 \cdot 100 = 49.2\%$

# Example: Shunt Motor - contd

- From previous it was shown that the torque for a shunt motor is proportional to the armature current, which has not changed, therefore the motor torque is still = 0.949Nm
- The new field current,  $I_f = 100/125 = 0.8A$
- From  $T = I_f I_a K_t$ , for  $I_f = 1A$  then  $K_t = 0.949/(1*1) = 0.949Nm/A^2$
- For the new value of  $I_f$ , the corresponding new value of the armature current can be calculated

$$I_a = 0.949/(0.8*0.949) = 1.25A$$

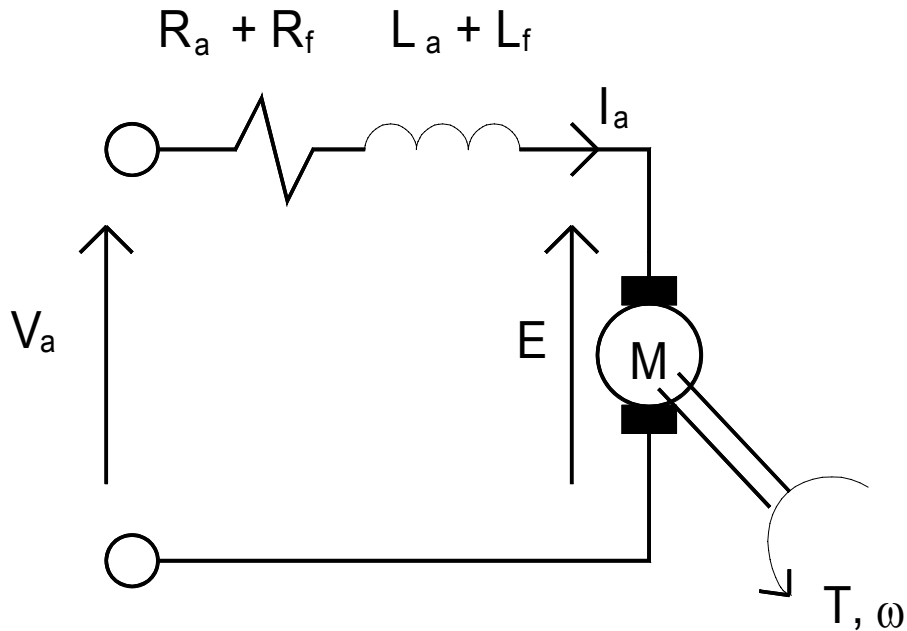
- Therefore,

$$\omega = \frac{V_a - I_a R_a}{I_f K_e} = \frac{100 - (1.25 * 1)}{0.8 * 0.955} = 129.25 \text{ rad / s} = 1234 \text{ rpm}$$

# Example: Shunt Motor - contd

- Say the load torque on the motor was trebled on sudden application of a new load. Investigate the effect on the motor speed for constant field current (with  $R_f = 100\Omega$ ).
- New torque,  $T = 3 * 0.949 = 2.847 \text{Nm} = I_f I_a K_t$
- Therefore new  $I_a = 2.847 / (1 * 0.949) = 3 \text{A}$  (This is intuitively correct as torque  $\propto I_a$  if the field current is constant)
- From 
$$\omega = \frac{V_a - I_a R_a}{I_f K_e} = \frac{100 - (3 * 1)}{1 * 0.955} = 101.57 \text{rad/s} = 970 \text{rpm}$$
- Therefore for a trebling of motor load the speed has changed by  $((990 - 970) / 990) * 100\% = 2\%$  i.e. almost constant speed with changing load
- Shunt motors are good for conveyors and machine tools which must operate at constant speed with varying loads

# Series DC Motor



- The flux,  $\Phi$ , of the motor is directly proportional to the field current,  $I_f$ , flowing to establish the magnetic field.
- For a series motor the current in the field and armature coils is exactly the same

$$T = I_f I_a K_t$$

$$= I_a^2 K_t$$

- From the circuit diagram,

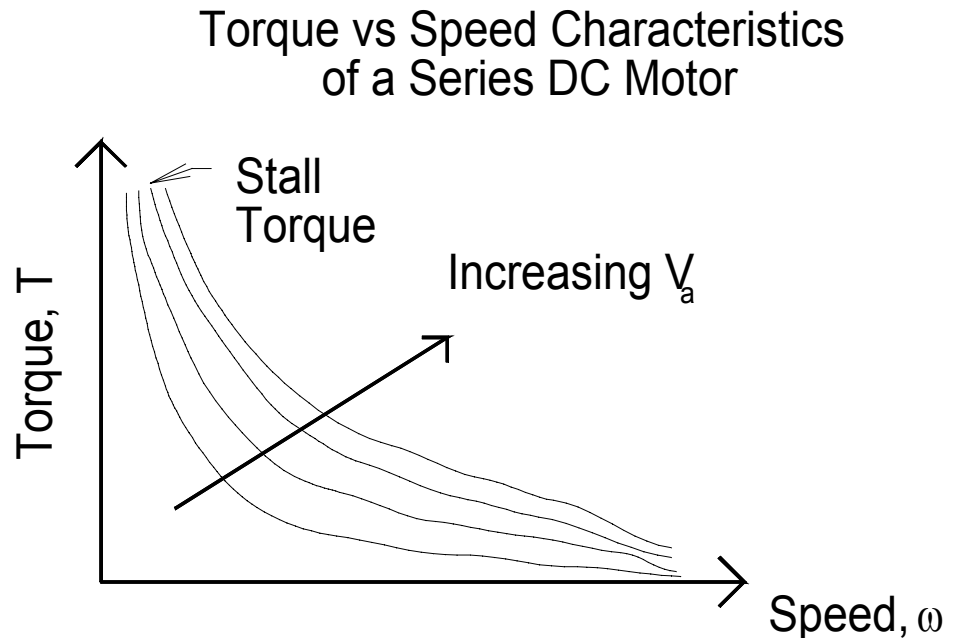
$$V_a = I_a(R_a + R_f) + E,$$

where  $V_a$  and  $I_a$  are average values and  $E = V_{emf} = \Phi \omega K_e$

- $E = I_a \omega K_e$

# Series DC Motor - contd

- At no load,  $T \sim 0$ ,  $I_a$  is minimal and  $E \sim V_a$ , and hence  $\omega$  is a maximum as  $E = I_a \omega K_e$
- As the load torque increases,  $I_a$  rises to the square power ( $T \propto I_a^2$ )
- As  $I_a$  increases  $E$  falls relative to  $V_a$ , however the speed falls according to  $E = I_a \omega K_e$
- Therefore as the torque increases the speed of a series motor decreases quite rapidly ( $\omega \sqrt{T} = \text{constant}$ )



# Practical Concerns: Series Motors

- From the torque vs speed characteristic it is clear that as the torque is decreased the motor speed increases - and will keep on increasing into overspeed (similar to potential field weakening run away for the shunt motor)
- It is good practice therefore to always have a load connected to a series DC motor before it is turned on to prevent overspeed. (generally this only applies to large DC motors as small motors tend to have enough friction to prevent this naturally)

# Speed Regulation

- Speed regulation determines the ability of the motor armature to maintain its speed when a changing load is applied
- (Analogous to transformer voltage regulation)
- Speed regulation is a ratio of **no-load** to **full-load** speed

$$\% \text{ speed regulation} = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\%$$

where  $n_{nl}$  is the no-load speed and  $n_{fl}$  is the full load speed in rpm

- The lower the speed regulation the more constant the motor speed will be over a range of applied loads

# Example: Series Motor

- Say the torque a series motor produces is 4Nm with 5A flowing in the armature. Calculate the value of the stall torque if the supply voltage to the motor is 100V and  $R_a = R_f = 1\Omega$  ?

- From previous it was shown that:  $T = I_a^2 K_t$

$$K_t = T/I_a^2 = 4/25 = 0.16 \text{ Nm/A}^2$$

- The armature current can be calculated by analysing the equivalent circuit:

$$I_a = \frac{V_a - V_{emf}}{R_a + R_f} = \frac{100 - 0}{1 + 1} = 50A$$

Note:  $V_{emf} = 0$  at stall i.e. zero speed

- Therefore the stall torque  $T_s = 50^2 * (0.16) = 400 \text{ Nm}$

# Example: Series Motor - contd

- From previous it was shown:  $V_a = I_a(R_a + R_f) + E$  and  $E = I_a \omega K_e$

$$K_e = \frac{V_e - I_a * (R_a + R_f)}{I_a \omega}$$

- This can be rearranged to give:
- From previous, if the motor were turning at 900rpm, with a current of 5A then the emf constant,  $K_e$ , is

$$K_e = \frac{100 - 5 * (1 + 1)}{5 * \left(\frac{900}{60} * 2 * \pi\right)} = 0.19 (V / A \cdot rad / s)$$

- If the motor now takes 20A calculate the new torque and motor speed?
- $T = I_a^2 K_t = 20^2 * 0.16 = 64 \text{Nm}$

$$\omega = \frac{V_a - I_a * (R_a + R_f)}{I_a K_e} = \frac{100 - 20 * (1 + 1)}{20 * 0.19} = 15.8 \text{rad / s} = 150.1 \text{rpm}$$

# Example: Series Motor - contd

- The speed regulation of the series motor is difficult to calculate as at full-load the motor is virtually stalling and at no-load it could overspeed.
- Most applications for DC series motors are traction drives for locomotives or for cranes, where a high starting torque, high accelerating torque and high speed at light load is required
- These applications need high starting torque to overcome the inertia of the load and the train requires to run at high speed once moving
- The series motor 'self-adjusts' to protect itself from overloads i.e. it will reduce speed as the load increases and then stall